Math 550
Vector Analysis
Fall 2001, USC

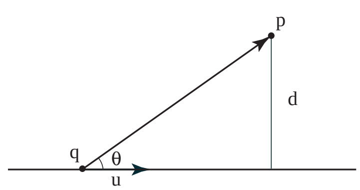
Sep 18, 2001

Practice Quiz 3

1. (Point to line) Show that the distance between a point p and a line ℓ in space is given by

$$\operatorname{dist}(p, \ell) = \frac{|\overrightarrow{qp} \times u|}{\|u\|},$$

where q is any point on ℓ, and u is a direction vector of ℓ.

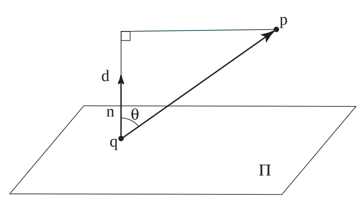


Hints: $d = \|\overrightarrow{qp}\| \sin\theta$, see the above figure.

2. (Point to plane) Show that the distance between a point p and a plane Π in space is given by

$$\operatorname{dist}(p, \Pi) = \frac{|\overrightarrow{qp} \cdot n|}{\|n\|},$$

where q is any point on ℓ, and u is a direction vector of ℓ.



Hints: $d = \|\overrightarrow{qp}\| \cos\theta$, see the above figure.

3. (Line to plane) Show that if a line ℓ does not intersect a plane Π, then the distance between them is given by

$$\operatorname{dist}(\ell, \Pi) = \frac{|\overrightarrow{qp} \cdot n|}{\|n\|},$$

where p is any point on ℓ, q is any point in Π, and n is a normal vector to Π.

Hints: Convince yourself that $\operatorname{dist}(\ell, \Pi) = \operatorname{dist}(p, \Pi)$.

4. (Plane to plane) Show that the distance between two parallel planes Π_1 and Π_2 is given by

$$\operatorname{dist}(\Pi_1, \Pi_2) = \frac{|\overrightarrow{p_1 p_2} \cdot n|}{\|n\|},$$

where p_1 and p_2 are any pairs of points of Π_1 and Π_2 respectively, and n is normal vector to Π or Π_2.

Hints: Convince yourself that $\operatorname{dist}(\Pi_1, \Pi_2) = \operatorname{dist}(p_1, \Pi_2)$.

4. (Line to line) Show that the distance between two *skew* lines ℓ_1 and ℓ_2 is given by

$$\operatorname{dist}(\ell_1, \ell_2) = \frac{|\overrightarrow{p_1 p_2} \cdot (u_1 \times u_2)|}{\|u_1 \times u_2\|},$$

where p_1 and p_2 are any pairs of points of and u_1 and u_2 are direction vectors for ℓ_1 and ℓ_2 respectively. (Skew means that the lines neither intersect, nor are parallel.)

Hints: Let Π_1 and Π_2 be planes which are orthogonal to $u_1 \times u_2$ and passing through p_1 and p_2 respectively. Convince yourself that $\operatorname{dist}(\ell_1, \ell_2) = \operatorname{dist}(\Pi_1, \Pi_2)$.