Math 241 Sep 13, 2000
Vector Calculus
Fall 2000, USC

PRACTICE QUIZ 2

Show that any parametric curve with constant speed, and the property that its acceleration vector is always parallel to its position vector, must be a circle.

Hints:

Let $\mathbf{r}(t)$, $\mathbf{v}(t)$, and $\mathbf{a}(t)$ denote, respectively, the position, velocity, and the acceleration vectors of the curve.

1. Recall that when the speed is constant, $\mathbf{v}(t)$ and $\mathbf{a}(t)$ are perpendicular (can you recall the proof?).

2. Then $\mathbf{v}(t)$ and $\mathbf{r}(t)$ must be perpendicular as well (why?).

3. Show that $(\|\mathbf{r}\|^2)' = 2\,\mathbf{r}(t)\cdot\mathbf{v}(t)$.

4. Conclude then that the magnitude of $\mathbf{r}$ is constant.