Math 23B
Multivariable Calculus
Fall 1999, UCSC

Nov 29, 1999

QUIZ 10

Time: 20min

---

1. Let C be a closed curve, and $\mathbf{v}$ be a constant vector. Show that

$$\int_C \mathbf{v} \cdot d\mathbf{s} = 0$$

in two different ways: (i) assume that C bounds some surface and apply the Stokes theorem; (ii) do a direct integration and apply the fundamental theorem of Calculus.

The next problem deals with an application of the above problem in differential geometry.

2 (Bonus). Let $\mathbf{c}(t)$ be a parameterization for C such that $\mathbf{c}'(t) \neq 0$. Then the unit tangent vectorfield of C is given by $\mathbf{t}(t) := \mathbf{c}'(t)/\|\mathbf{c}'(t)\|$. The *tantrix* of C, which we denote by T, is the curve in the unit sphere which is traced by $\mathbf{t}(t)$. Use the above problem to show that if C is a closed curve then T must intersect every great circle in the unit sphere.

Hint: It is enough to show that, for every constant vector $\mathbf{v}$, $\mathbf{t}(t) \cdot \mathbf{v}$ is either equal to zero for all t, or else changes sign.

Each problem is worth 10 points.