Math 23B
Multivariable Calculus
Fall 1999, UCSC

Nov 16, 1999

PRACTICE QUIZ 3

1. Use Green's theorem to show that the area of a region $D \subset \mathbf{R}^2$ is given by

$$A = \frac{1}{2} \int_{C^+} x dy - y dx,$$

where C^+ denotes the boundary of D oriented counterclockwise.

Hints:

(i) Recall Green's theorem: $\int_{C^+}(Pdx + Qdy) = \int\int_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dxdy$.

(ii) Let $P := -y$, and $Q := x$.

2. Use the previous problem to compute the area of the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Hint: Show that $(a\cos t, b\sin t, 0)$, $0 \leq t \leq 2\pi$, gives a counterclockwise parametrization for the ellipse.

3. Find a clockwise parameterization for the ellipse and repeat problem 2. How does your answer differ from the previous problem?