Math 23B
Multivariable Calculus
Fall 1999, UCSC

Nov 2, 1999

MIDTERM

Time: 105min

1. Compute the volume of a right circular cone with height h and base radius R. (**Bonus**: do this problem in two different ways: (i) by using Cavlieri's principle, and (ii) by evaluating a double integral.)

2. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Compute the average value of the function $f(x, y) := e^{x^2+y^2}$ over the disk $x^2 + y^2 \leq 1$.

4. Evaluate $\int\int_D x\, dA$, and $\int\int_D y\, dA$, where D is the triangle bounded by $x = 0$, $y = 0$, and $x + y = 1$. (**Bonus**: what is the value of these integrals if D is reflected with respect to the y-axis?)

5. Find the center of mass of half a ball of radius R.

6. Compute the length of the helix given by $c(t) := (\cos t, \sin t, t)$, $0 \leq t \leq 2\pi$. (**Bonus**: what is the average value of the square of the distance of this curve from the origin?)

7. Compute the total work done by a particle moving along the cubic curve, $c(t) := (t, t^2, t^3)$, $0 \leq t \leq 1$, inside the vector field $F(x, y, z) := (y, x, 2)$. (**Bonus**: solve this problem in two different ways).

8. Find the total mass of the region inside the sphere $x^2 + y^2 + z^2 = 1$, and above the cone $z = \sqrt{x^2 + y^2}$, if the density is given by $\delta(x, y, z) := \sqrt{x^2 + y^2 + z^2}$.

Each problem is worth 10 points.
The bonuses are worth an extra 5 points each.

LaTeX ..... $\mathcal{MG}$