

MIDTERM

Time: 70min

1. Integrate:

a) $\int_0^{\pi/2} \cos^3 \theta \, d\theta$

b) $\int_1^e (\ln x)^2 \, dx$

c) $\int \frac{x}{x^2 - 5x + 6} \, dx$

d) $\int \sqrt{4 - x^2} \, dx$

2. a) Set up an integral corresponding to the area of the region bounded by the curves: $y = \sqrt{1 + x^3}$, $y = 1 - x$, and $x = 2$. b) Find a numerical estimate for this integral using a Reimann sum with 4 subdivisions.

3. Compute the volume of a sphere of radius r .

4. Suppose that, on a certain day, from noon to 3 pm the average temperature was 60° , from 3 to 4 pm the average temperature was 48° , and from 4 to 6 pm the average temperature was 30° . Use integrals to find the average temperature from noon until 6 pm.

5. A circular swimming pool has diameter 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work is required to pump all of the water over the side? (Suppose, for convenience, that water weighs 1 lb/ft³, and also that the gravitational constant $g = 1$).

The first problem is worth 40 points (10 points for each part), and the rest are 15 points each.