

# Midterm 3

Time: 60min

Note: Justify all your answers.

1. Find  $dy/dx$  in *one* of the following:

a)  $y = \frac{\sqrt{x+13}}{(x-4)(\sqrt[3]{2x+1})}$

b)  $y = \tan^{-1} x$

2. Find *two* of the following integrals

a)  $\int \frac{1}{x^2 + 2x + 10} dx$

b)  $\int \cos^5 x dx$

c)  $\int \tan^{-1} x dx$

3. Find *one* of the following limits

a)  $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}$

b)  $\lim_{x \rightarrow 0} (x^2 \ln x)$

4. Write the following number as as the ratio of two integers.

3.1222222...

5. Determine whether or not the following series converge.

a)  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots$

b)  $\sum_{n=2}^{\infty} \frac{n}{\ln n}$

c)  $\sum_{n=1}^{\infty} \frac{n+7}{n^2 \sqrt{n}}$

d)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n!}$

6. Find the convergence set of the power series

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

**7 (Bonus).** Choose one of the following problems:

- a) Find the sum of the alternating harmonic series (*Hint*: Find a power series for  $\ln(x + 1)$  by integrating the power series for  $1/(1 + x)$ ).
- a) Find an infinite series which converges to  $\pi$  (*Hint*: Find a power series for  $1/(1 + x^2)$  and integrate it to find a power series for  $\tan^{-1} x$ ).
- c) Find  $\lim_{n \rightarrow \infty} \frac{x^n}{n!}$  (*Hint*: consider the series  $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ ).

*Problems 2 and 5 are worth 20 points and 40 points respectively; the rest are worth 10 points each*