MIDTERM 1 **Time: 50min**

1. Find the following limits. Justify your answers.

$$\textbf{a)} \lim_{x \to \infty} \frac{\cos x}{x} \qquad\qquad \textbf{b)} \lim_{x \to 0^-} \frac{x}{|x|}$$

$$\textbf{c)} \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \qquad\qquad \textbf{d)} \lim_{x \to \infty} \frac{\sqrt{2x + 1}}{x + 4}$$

2. Find $f'(x)$, if $f(x) = \cot x = \frac{\cos x}{\sin x}$.

3. Use the $\epsilon - \delta$ definition of limit to prove that $\lim_{x \to 4}(3x - 7) = 5$. How close to 4 must we choose x so that $3x - 7$ is within 0.01 of 5?

4. Does $x^5 + 4x^3 - 7x + 14 = 0$ have any real roots? Justify your answer.

5. Use definition of derivative to find the derivative of $f(x) = \frac{1}{x}$.

6. Find the equation of the tangent line to $y = x^2 - 2x + 2$ at the point $(1, 1)$.

7 (Bonus). Suppose that a car travels a total distance of 100 miles, reaching a top speed of 100 miles/hr at some point during the journey. Show that there must be a time when the distance travelled by the car is exactly equal to its speed at that moment.

Problems 2, 4, and the bonus are worth 10 points each. The rest are 20 points each.

LaTeX .... $\mathcal{MG}$