

FINAL EXAM

Time: 3hrs

1. Evaluate (a) $\int \ln x \, dx$, (b) $\int \frac{e^t}{1+e^t} \, dt$.
2. Compute the area bounded by a circle of radius r .
3. Calculate the energy required to move an object infinitely far from the surface of the earth, and find the escape velocity.
4. Show that $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots)$.
5. Suppose a ball is dropped from an initial height of 10 ft, and each time it bounces, it rises to $\frac{3}{4}$ of the previous height. (a) What is the total vertical distance traveled by the ball? (b) When will the ball come to a stop?
6. (a) If you deposit money continuously at a constant rate of \$1000 per year into a bank account that earns 5% interest, how many years will it take for the balance to reach \$10,000? (b) How many years would it take if the account had \$2000 in it initially?
7. Find a formula for the volume of a cone of height h and base radius r .
8. For what values λ is $y = Ax^\lambda$ a solution of the equation
$$x^2y'' + 2xy' - 6y = 0?$$
9. Suppose the average value of a function, $f(x)$, over the interval $[a, c]$ is equal to 1, and over the interval $[c, b]$ is equal to 2. What is the average of $f(x)$ over the interval $[a, b]$.
10. Suppose a car moves with increasing velocity $v(t)$ from time $t = a$ to $t = b$. (a) Find an upper and lower estimate for the total distance traveled by dividing the interval $[a, b]$ into n equal segments. (b) What is the limit of these estimates as n goes to infinity? Prove your answer.

Each problem is worth 10 pts.