110.302 February 18, 2000
Differential Equations
Summer 1994, JHU

TEST I

Time: 50min

1. Find the general solution of the following two differential equations:

$$x^2y' + y^2 = 0, \qquad e^{-x^2}y' = x.$$

2. Find the orthogonal trajectories to the family of curves given by $y = Ce^x$ and sketch both families.

3. Uranium-238 decays at a rate proportional to the amount present. If x_1 and x_2 grams are present at times t_1 and t_2 show that the half-life is:

$$\frac{(t_2 - t_1)\log 2}{log(x_1/x_2)}.$$

4. Three vertices of a rectangle of area A lie on the x-axis, y-axis, and at the origin. If the forth vertex moves along the curve y = f(x) in the first quadrant in such a way that the rate of change of A with respect to x is proportional to A, find the equation of the curve.

5. (Extra Credit) A mass m is thrown up from the surface of the earth with initial velocity v_o. If air resistance is assumed to be proportional to the velocity, with constant of proportionality k, and if the only other force acting on the mass is constant gravitational force, find an expression for the maximum height attained.

each problem is worth 10 pts.

LaTeX ..... $\mathcal{MG}$